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"Transport of Nonneutral Plasmas"

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The research program on "Transport of Nonneutral Plasmas" includes both on-going experiments and theory on transport in pure electron plasmas; and the design, construction and operation of a new pure ion plasma apparatus. Both aspects of this program are proceeding well, as detailed below and in the scientific publications referenced herein.

Recent experiments on the electron apparatus have elucidated the transport and turbulence associated with diocotron instabilities and vortex structures, and have measured the vortex pairing instability in detail. Recent theory work has analyzed the newly observed $l = 1$ diocotron instability, analyzed "negative temperature" 2D thermal equilibria, and investigated the crystalline equilibria of cold ion clusters. The characteristics of the new ion apparatus have been determined, the major components have been designed, and purchasing and construction is underway. We expect to have the new apparatus operational as planned near the end of the second year of this grant.

We have been studying the transport which results from the essentially two-dimensional $\mathbf{E} \times \mathbf{B}$ drift of plasma across field lines. These motions are the fastest radial transport process that can take place in our devices. The electron dynamics can be approximated by 2-dimensional guiding center theory. Here, the axial bouncing of the individual electrons averages over any z-variations at a rate fast compared to $r-\theta$ motions. In this approximation, the 2D drift-Poisson equations for the evolution of the electron column are isomorphic to the 2D Euler equations for an inviscid fluid of uniform density ρ.[9] In the electron system, the flow vorticity Ω is proportional to the density n, which we measure directly.

Thus, in this approximation an initial distribution of electrons $n(r,\theta,t=0)$ in a cylinder with vorticity $\Omega \propto n$ will evolve exactly the same as an initial distribution of

vorticity Ω in a uniform fluid such as water. The electron experiments have many advantages for testing 2D fluid theory: the magnetic field tends to keep the electron system 2D, and the electron column has low internal viscosity and has no boundary layers.

Diocotron Instabilities

When the charge density profile $n_0(r)$ is non-monotonic (i.e. at least partially "hollow"), unstable diocotron modes may appear. These modes vary as $\delta n(r) \exp\{ikz + il\theta - i\omega t\}$, with $k=0$ for the 2D modes considered here. Experimentally, the unstable modes are found to be distinct from the stable modes.[5,9] That is, for any given l, the stable and unstable modes coexist, with distinct frequencies and radial eigenfunctions. We observe instabilities with $l \geq 2$, as expected. Furthermore, we observe a robust exponential instability for $l=1$ where none is predicted. These instabilities give rise to large θ-asymmetries in $n(r,\theta)$ and $\phi(r,\theta)$, which then result in radial transport on the drift motion time scale. Our data for $n(r,\theta,t)$ characterizes the $k=0$ modes rather completely.

For $l=1$, we are able to observe exponential mode growth over a range of up to 2 decades before nonlinear saturation occurs.[5,9,22,27] The instability appears to be a linear process, i.e. independent of the perturbation amplitude at $t=0$. Typically, the instability e-folds in about 6 rotation periods for the column.

Prior theory had concluded that there are no instabilities for $l=1$, whereas recent theory work has elucidated an instability which grows algebraically with time.[6,26] The theoretical growth with time can be obtained by numerically integrating a linear Laplace transform solution which starts from initial conditions corresponding to the experiments. Alternately, one can perform an asymptotic analysis on the Laplace transform solution, obtaining a perturbation proportional to $t^{1/2}$ as $t \to \infty$. Although the

differences between experiment and theory here are large, there are intriguing similarities. The early time instability growth is seen to be similar. Further, the time-asymptotic theory perturbation is self-shielding,[6] as is observed experimentally at all times.[5] These experimental and theoretical results on the new $l=1$ instability on hollow columns have recently been published as two *Physical Review Letters*,[5,6] and as contributed talks at APS/DPP-89,[22] and at APS/DFD-89.[26,27]

For $l=2$, an exponentially unstable mode is observed for a wide range of hollow profiles,[1,2] and these results are basically consistent with 2D theory predictions. Numerical solution of the eigenvalue equation for realistic smooth profiles has given fair agreement with unstable $l=2$ growth rates in these preliminary experiments. The $l=2$ perturbation grows exponentially until the mode is fully saturated and forms two vortex structures in a background of lower vorticity. The two vortices persist for many rotations, then merge to the center. Presumably, the two-vortex state is unstable, and the merger is the result of a vortex pairing instability. There may be extended filamentary structures arising during the vortex formation and disintegration, giving rise to density fluctuations on smaller spatial scales.

This work in instability-driven transport has been the subject of several invited[14,18] and contributed talks,[20] and has been published in the conference proceedings of the IAEA[1] and a US-Japan workshop.[2]

Vortex Dynamics

The two vortex merger process can be studied as an isolated process, independent of any diocotron instability process. In a sense, this is a generalization of our recent work on the (stable) nonlinear $l=1$ diocotron mode,[7] which may be thought of as a single vortex interacting with a wall. Straightforward manipulation techniques allow us to form an initial condition consisting of two electron columns of chosen profile and

placement. Here, we consider the particular case of two equal columns which are placed symmetrically on either side of the cylindrical axis, separated by a distance 2D.

We observe that the behavior of the two vortices depends dramatically on their separation to diameter ratio.[21,25] If the vortices are separated by more than 2.0 diameters, we observe that they orbit around each other relatively unperturbed for up to 10^4 orbits. If the vortices are initially separated by 1.9 diameters, their mutual interaction quickly results in filamentary tail formation, but the vortices still orbit around each other for about 100 orbits before merging at the center. If they are separated by 1.8 diameters, we observe merger at the center in less than one orbit period. Thus, the time to merge abruptly increases from 10 μsec to about 1 sec as the separation varies from 1.8 to 2.0 diameters.

This may represent the cleanest experimental measurement to date of this fundamental vortex interaction process. The low inherent viscosity of the plasma and the total absence of boundary layers at the wall both contribute to absence of secondary effects which could obscure its result. These preliminary results are in fair agreement with theory and computational results, and with an experiment in a water tank at Australian National University.

The orbit frequency and vortex shape distortion can also be measured as a function of separation of the two vortices. We find that the orbit frequency is well-modelled by the interaction of 2 "point" vortices of appropriate total circulation (*i.e.* total charge per unit length axially), but that the wall interaction must be included for large separations. We also find that the interacting vortices elongate towards each other even in the absence of merger. The elongation appears roughly consistent with various "moment models" and numerical calculations, but the experimental data has not yet been analyzed in detail.

Of course, 2 vortices may merge even if they are not symmetric in size or placement. Experimentally, the asymmetry is easily controlled. Also, it is easy to take 3 vortices as the initial condition. Here, the parameter space is substantially larger, and chaotic behavior is often observed.

This work on vortex dynamics has been presented at Plasma Physics[21] and Fluids Dynamics conferences,[25] and is being prepared for publication.

Negative Temperature Equilibria

Theoretically, one candidate for the configuration resulting from vortex mergers or violent shear-flow instabilities is the "most probable state" obtained by maximizing the Vlasov entropy $S = -\int n \ln n$ of the distribution of guiding centers, subject to the constraints imposed by certain conservation laws. In the continuum limit, the integral of any function of density is conserved. Nonideal dissipation or coarse-graining eliminates the small scales of turbulence, and breaks the conservation of all but the simplest of these functionals. Hence only a few gross conservation laws may survive.

We have analytically studied the equilibria which result from taking the total charge, the canonical angular momentum, and the electrostatic energy to be the only conserved quantities.[3,8,23,28] (This assumes that the guiding-center degrees of freedom are decoupled from velocity space over the time scales which characterize turbulent relaxation.) The constrained maximum-entropy configurations are then guiding-center thermal equilibria, described by the equation

$$n(r,\theta) = n_0 \exp\{-\beta(e\phi - \omega r^2)\} \quad , \tag{1}$$

where the parameters n_0, β, and ω are determined by the charge, energy, and angular momentum.

We find that "negative temperature" solutions exist for certain parameter regimes and that these equilibria are displaced from the geometrical center of the system at large energies or angular momenta. Such an off-axis state can be regarded as a dynamic equilibrium or equivalently as a finite-amplitude diocotron mode. $1/\beta$ is an effective temperature, and is negative for the off-axis equilibria. These results have been confirmed by Monte Carlo simulations of ensembles of guiding centers. We find that the statistical fluctuations can be surprisingly large, much as in a system undergoing a phase transition. However the relation between these thermal equilibria and the final states observed in experiments is not yet clear.

These off-axis equilibrium results have been discussed at conferences,[23,28] and have been published in *Phys. Rev. Lett.*,[3] with a more complete description submitted to *Phys. Fluids*.[8]

Ion Crystals

When a single species plasma in thermal equilibrium is cooled to a sufficiently low temperature, it passes first into the liquid state and then into a crystalline state. The theory of these states is well understood for infinite sized systems with no boundaries: this system is called the One Component Plasma (OCP). Computer simulations and analytic theory for an infinite homogeneous OCP predict that for $\Gamma \gtrsim 2$ the system of charges begins to exhibit local order characteristic of a liquid, and for $\Gamma \gtrsim 172$ there is a first-order phase transition to a bcc crystal.

However, the physics is substantially more complicated for realistic clusters of $10^2 - 10^4$ particles. In cryogenic electron plasma experiments, $\Gamma \sim 1$–2 have been achieved[4]; in ion plasma experiments at NIST, Γ values in the range of several hundred have been measured, putting the system well into the regime of strong correlation.

We have studied the properties of these finite-sized clusters using a combination of analytic theory and computer simulation.[4,11,13] For moderate values of Γ ($2<\Gamma\lesssim 100$) we find that the density of the cloud exhibits spatial oscillations. These oscillations have maximum amplitude at the cloud surface and decay back to the background density n_0 with increasing distance from the surface. The oscillations are evidence of local order, and the damping length is a measure of the correlation length. For $\Gamma\gtrsim 140$, the cloud separates into concentric spheroidal shells. In this regime charges rarely move from shell to shell, but they diffuse freely within the shell surfaces. Thus, the system might be characterized as a liquid within the shells, and as a solid in the direction perpendicular to the shells, as in a smectic liquid crystal. For still larger values of Γ particle diffusion within the shells also approaches zero and an imperfect 2-D hexagonal crystal is formed within each shell.

We find that the lattice structure in the crystalline regime is quite different from the bcc lattice predicted for an infinite homogeneous OCP. Simulations have been performed on up to 2000 ions, and no evidence of bcc structure has been observed. Recent theoretical work based on the extremely oblate "slab" limit of the bounded crystal predicts that the system may have to be quite large before the minimum energy state displays bcc symmetry, perhaps requiring as many as 60 shells before the infinite homogeneous structure is recovered.[11] Thus, the addition of a surface term to the free energy can change the thermal equilibrium lattice structure, even if this surface term is relatively small compared to the bulk term.

Recently, the free energy of the classical one-component plasma has been calculated analytically in the crystalline phase for both fcc and bcc lattices to $O(T^2)$, where T is the temperature.[4] By application of thermodynamic perturbation theory, we can explicitly evaluate the effect of three and four phonon interactions on the partition function. Periodic boundary conditions are applied to make contact with previous

numerical work, in which the $O(T^2)$ term was assumed to be negligible. We find that this term is much larger than previously thought. This increases the thermodynamic stability of the crystal phase over previous estimates.

These results have been the subject of several invited[15,16,19] and contributed talks,[34] and have been published in *Physical Review A*[4,10] and two Conference Proceedings.[11,13]

Ion Apparatus Development

We are presently developing a new apparatus to contain a nonneutral ion plasma suitable for laser induced fluorescence and optical tagging diagnostics. The goal is to obtain detailed understanding of the transport of particles across magnetic field lines in various parameter regimes. The characteristics of the new apparatus have now been determined, laboratory space is being renovated for this use, bids have been obtained on the major components, and other mechanical and electronic components are being designed and constructed. At present, we foresee no insurmountable difficulties in the development of this apparatus. The main uncertainty at present is in acquisition of the laser system: the ONR did not provide funds for this purpose, but we hope to make a start on the laser diagnostics using cost savings from other categories.

Laser induced fluorescence (LIF) has been successfully used in plasma physics for several years. The use of this technique has produced scientific results on the fundamental plasma physics of gas discharges and alkaline plasmas and also in Tokamak research. However, we are in a parameter region significantly different than the existing experiments. Our ions will be in the same temperature range as that of gas discharge or a Q-plasma, but the density is about two orders of magnitude smaller, leading to a smaller number of detected photons. We intend to use Magnesium 24 as the ion, with plasma parameters of $n_i \geq 10^7\,\mathrm{cm}^{-3}$ and $T_i = 0.1 - 10\,\mathrm{eV}$.

In designing the LIF system, we have considered a number of effects which complicate the signal, including Doppler broadening, power broadening, Zeeman splitting, isotopic shifts, hyperfine structure, and transit time effects. While restrictive, none of these complications present insurmountable problems.

For optical tagging, we need to optically pump ions into a state which has a long lifetime compared to the physical time scale of the investigated process. A search laser is then used to detect the test particles at different spatial locations in the plasma. In addition to spatial information, information about velocity space is also obtained if the laser band-width is narrower than the Doppler band-width. Two kinds of long lifetime states have been used previously: electronic metastable states and electronic spin states. Since the lifetime of the metastable ion state is generally limited to several tens of seconds, we are going to use the electronic spin state. Since these spin states have a lifetime which is probably limited only by spin exchange, the lifetime should be sufficient for our purposes when the ion temperature is smaller than 1 eV. We calculate that we will need typically a 90 μs pulse with a 1 mW CW single mode laser to flip all the spins along the laser beam path inside of the plasma.

Magnesium-24 was selected for the following properties: 1) an optical transition from the ground state in the visible region or near UV; 2) no long lifetime metastable states which could quench the fluorescent signal; 3) no hyperfine structure which would complicate the LIF signal; 4) a non-integer total electron spin allowing us to make use of spin state tagging technique. For ^{24}Mg, the interesting transitions (e.g. $3s\,^2S_{1/2} \rightarrow 3p\,^2P_{1/2}$) are at about 280 nm.

With this choice, the anticipated plasma parameters are as follows:

Magnesium plasma, singly ionized	
Magnetic field	$B \sim 6$ T
Density	$n \geq 10^7$ cm^{-3}
Brillouin limit	$n_B \sim 4 \times 10^9$ cm^{-3}

Ion temperatures	$T_i \sim 0.1 \to 10$ eV
	$\Delta\nu_{\mathrm{Doppler}}$ $5.3 \to 53.$ GHz
Plasma diameter	$2R_p \sim 2.$ cm
Plasma length	$L_p \sim 25. \to 50.$ cm
Confinement time	$\tau \sim$ several hours
Debye length (1 eV)	$\lambda_D \sim 0.23$ cm
Cyclotron frequency	$\nu_{ci} \sim 3.8$ MHz
Plasma frequency	$\nu_{pi} \gtrsim 135$ kHz
Rotation frequency	$\nu_r \gtrsim 2.4$ kHz
Bounce frequency	$\nu_b \lesssim 4$ kHz
Ion collision frequency (90°)	$\nu_{ii} \sim 0.5 \to 10$ Hz
Base pressure	$p \leq 10^{-10}$ Torr
Ion neutral collision (90°)	$\nu_{in} \leq 10^{-4}$ Hz

We need an intense tunable narrow band light source. It appears that the only possibility is a dye laser, since the only other sufficiently tunable laser is solid state Ti-Sapphire at 700 nm $\to$ 980 nm. No existing dye works at 280 nm, so we will use a dye at 560 nm and frequency double. Our requirements for the laser are the following:

Tunable and stabilized;

Computer controlled with capability for measuring the laser wavelength;

Frequency doubled;

Band-width much smaller than the Doppler broadening;

Laser power "sufficient."

We believe that these requirements are best met by a CW laser with external frequency doubling, giving a power of 1 mW at 280 nm.

The characteristics of the optical transport and fluorescence detection system have been analyzed. Considering the solid angle of collection, grid and fiber optic losses, vacuum window, filter and lens losses, and photomultiplier tube quantum efficiency, we expect an overall efficiency of light collection around 1.5×10^{-4}. For our plasma

parameters, this results in signal-to-noise ratios of 10 or better for a typical experiment when integration times of 1 msec are used.

The room-temperature bore superconducting magnet is perhaps the most basic (and expensive) single part of the experiment. However, the characteristics of the magnet could not be specified until the laser diagnostic characteristics were understood and the test ion selected. For example, the mass of the ion determines the magnetic field required for a given ion density. Similarly, the size of the magnet is determined by the ion Larmor radius, the ion Debye length, the optical access requirements, etc. With these questions answered, we have specified the magnet and received bids from vendors. Our specified magnet has a bore of 8 3/4", a uniform field length of 60 cm, and a field strength of 6 Tesla. We expect delivery of the complete magnet system by December 1990.

The laboratory layout was dependent on the specification of the magnet, since there are numerous safety and practical concerns with a magnet of this strength and size. We have designed a workable laboratory, and renovations funded by UCSD should be completed by August 1990.

At present, we intend to obtain the $^{24}Mg^{+}$ ions from a vacuum arc source. The plasma produced by the source will probably have a substantial level of impurities, an unacceptably high temperature, density jitters of about 20%, and the wrong radial profile. We intend to get rid of the unwanted ions by heating with RF at their cyclotron frequency, increasing their larmor radius until they hit the wall. We intend to cool the ion plasma by collisions with neutral gas such as He. If a fast magnesium ion has a head-on collision with a helium neutral which is at rest, it loses 25/49 of its energy. Thus in 10 collisions a 250 eV plasma would cool to room temperature. This requires about 10 seconds at a pressure of 2×10^{-7} Torr. We have performed tests on the effects of puffed gas loads on UHV pumps, and feel that this neutral load is

acceptable, especially if the pump is valved off during the gas puff. Finally, we intend to manipulate the plasma density profile and total particle number by well-established techniques such as "electric tilt" perturbations and selective axial release of particles. These various manipulations will be done with computer sensing and control. This is a straightforward extension of techniques in use on present apparatuses. Although there may be various difficulties in the development of this new experiment, we are optimistic that the new ion machine will be in operation on schedule.

RECENT ONR-SUPPORTED PUBLICATIONS

Journal Articles

1. C. F. Driscoll, J. H. Malmberg, K. S. Fine, R. A. Smith, X.-P. Huang and R. W. Gould, "Growth and Decay of Turbulent Vortex Structures in Pure Electron Plasmas," in *Plasma Physics and Controlled Nuclear Fusion Research 1988*, Vol. 3, 507-514. Vienna: IAEA (1989).
2. C. F. Driscoll, R. A. Smith, X-P. Huang and J. H. Malmberg, "Growth and decay of vortex structures in pure electron plasmas," in *Structures in Confined Plasmas—Proc. of Workshop of U.S.-Japan Joint Institute for Fusion Theory Program*, Report No. NIFS-PROC-2, 69-76. Nagoya: National Institute for Fusion Science (1990).
3. R. A. Smith, "Phase-Transition Behavior in a Negative-Temperature Guiding-Center Plasma," *Phys. Rev. Lett.* **63**, 1479 (1989).
4. D. H. E. Dubin, "Correlation Energies of Simple Bounded Coulomb Lattices," *Phys. Rev.* **A40**, 1140 (1989).
5. C. F. Driscoll, "Observation of an Unstable $l = 1$ Diocotron Mode on a Hollow Electron Column," *Phys. Rev. Lett.* **64**, 645 (1990).
6. R. A. Smith and M. N. Rosenbluth, "Algebraic Instability of Hollow Electron Columns and Cylindrical Vortices," *Phys. Rev. Lett.* **64**, 649 (1990).
7. K. S. Fine, C. F. Driscoll and J. H. Malmberg, "Measurements of a Nonlinear Diocotron Mode in Pure Electron Plasmas," *Phys. Rev. Lett.* **63**, 2232 (1989).
8. R. A. Smith and T. M. O'Neil, "Nonaxisymmetric Thermal Equilibria of a Cylindrically Bounded Guiding Center Plasma or Discrete Vortex System," submitted to *Physics of Fluids B* (1990).
9. C. F. Driscoll and K. S. Fine, "Experiments on Vortex Dynamics in Pure Electron Plasmas," To appear in *Phys. Fluids B* (1990).
10. D. H. Dubin and T. M. O'Neil, "First Order Anharmonic Correction for the Free Energy of a Coulomb Crystal in Periodic Boundary Conditions," accepted by *Phys. Rev. A*.
11. D. H. E. Dubin and T. M. O'Neil, "Theory of Strongly-Correlated Pure Ion Plasmas in Penning Traps," in *Strongly Coupled Plasma Physics,* (S. Ichimaru, editor), Elsevier Science Pub. B.V./Yamada Science Foundation, 189-200 (1990).
12. C. F. Driscoll, "Wave and Vortex Dynamics in Pure Electron Plasmas," to appear in AIP Conference Proceedings series.
13. D. H. E. Dubin and T. M. O'Neil, "Pure Ion Plasmas, Liquids and Crystals," to appear in AIP Conference Proceedings series.

Invited Papers

14. C. F. Driscoll, "Experiments on Vortex Dynamics in Pure Electron Plasmas," *Bull. Am. Phys. Soc.* **34**, 2001 (1989).

15. D. H. E. Dubin and T. M. O'Neil, "Theory of Strongly-Correlated Pure Ion Plasmas in Penning Traps," Yamada Conference on Strongly Coupled Plasma Physics, Tokyo, Japan (1989).
16. D. H. E. Dubin, "Strongly Correlated Trapped Pure Ion Plasmas," to appear in *Proc. of 1989 International Conference on Plasma Physics*, New Delhi, India (1989).
17. T. M. O'Neil, "Plasmas with a Single Sign of Charge (A Review of Recent Theory and Experiment," Sherwood Theory Conference, Williamsburg, VA (1990).
18. C. F. Driscoll, "Wave and Vortex Dynamics in Pure Electron Plasmas," Topical Conference on Research Trends in Nonlinear and Relativistic Effects in Plasmas, La Jolla, CA (1990).
19. D. H. E. Dubin and T. M. O'Neil, "Pure Ion Plasmas, Liquids and Crystals," Topical Conference on Research Trends in Nonlinear and Relativistic Effects in Plasmas, La Jolla, CA (1990).

Contributed Papers

20. X.-P. Huang, C. F. Driscoll and J. H. Malmberg, "Dynamical and Statistical Measurements on Vortex-Driven Turbulence," Plasma Div., APS, Anaheim, *APS Bull.* 34, 1931 (1989).
21. K. S. Fine, C. F. Driscoll, T. B. Mitchell and J. H. Malmberg, "Dynamics and Instabilities of 2D Electron Plasma Vortices," Plasma Div., APS, Anaheim, *APS Bull.* 34, 1932 (1989).
22. C. F. Driscoll, "Observation of an $l=1$ Diocotron Instability on Hollow Electron Columns," Plasma Div., APS, Anaheim, *APS Bull.* 34, 1932 (1989).
23. R. A. Smith and T. M. O'Neil, "Nonaxisymmetric Thermal Equilibria of a Non-neutral Guiding Center Plasma in a Cylindrical Trap," Plasma Div., APS, Anaheim, *APS Bull.* 34, 1934 (1989).
24. D. H. E. Dubin, "Structure of a Bounded Coulomb Lattice," Plasma Div., APS, Anaheim, *APS Bull.* 34, 1934 (1989).
25. K. S. Fine, C. F. Driscoll, T. B. Mitchell and J. H. Malmberg, "Dynamics and Instabilities of 2D Electron Plasma Vortices," Fluid Dynamics Div., APS, Palo Alto, *APS Bull.* 34, 2308 (1989).
26. R. A. Smith and M. N. Rosenbluth, "Algebraic Instability of Cylindrical Shear Flows," Fluid Dynamics Div., APS, Palo Alto, *APS Bull.* 34, 2336 (1989).
27. C. F. Driscoll, "Observation of an $l=1$ Shear Instability on Hollow Electron Columns," Fluid Dynamics Div., APS, Palo Alto, *APS Bull.* 34, 2336 (1989).
28. R. A. Smith and T. M. O'Neil, "Maximum Entropy Equilibria of a Nonneutral Guiding-Center Plasma," Sherwood Theory Conference, Williamsburg, VA (1990).